Magnetically induced pumping and memory storage in quantum rings

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Nanoscopic rings pierced by external magnetic fields and asymmetrically connected to wires behave in sharp contrast with classical expectations. By studying the real-time evolution of tight-binding models in different geometries, we show that the creation of a magnetic dipole by a bias-induced current is a process that can be reversed: connected rings excited by an internal ac flux produce ballistic currents in the external wires. In particular we point out that, by employing suitable flux protocols, single-parameter nonadiabatic pumping can be achieved, and an arbitrary amount of charge can be transferred from one side to the other. We also propose a set up that could serve a memory device, in which both the operations of *writing* and *erasing* can be efficiently performed.

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I. INTRODUCTION

The periodic modulation of the parameters of a mesoscopic system can generate a finite dc current in absence of an applied bias[1]. This phenomenon is known as quantum pump effect. The quantum pumping has attracted considerable attention during the last decade since it may provide a novel way to define a better current standard[2], to produce spin currents[3], and to realize memory storage[4]. In a seminal work Brouwer[5] has shown that in case of adiabatic modulations at least two parameters (such as gate voltages) are needed to obtain a non-vanishing pumped current. The connection of his arguments with the Berry phase is well understood[6]. An intriguing challenge in this field involves, however, the incorporation of non-adiabatic effects. Simple schemes for pumping beyond the adiabatic approximation in Floquet space were provided in Refs. 7 and 8 . In this space, the phase difference between two parameters was shown to provide a way to accumulate a nonvanishing phase shift along a closed trajectory in a way that is analogous to a magnetic flux. This motivated the proposal of a quantum pump with a single time-dependent potential and where the left-right symmetry is broken by a magnetic field[8]. In Ref. 9 an alternative method based on real-time simulations was proposed to study nonadiabatic effects on the pumping properties of nanoscale junctions.

In the present study we predict the feasibility of single-parameter non-adiabatic pumping in a circuit containing a laterally connected ballistic ring pierced by a time-dependent magnetic field[10]. We show that some aspects of this finding are unexpected semiclassically but can be rationalized in terms of a recent theory[11] of current-induced magnetic moments of quantum rings. As a further nonconventional application of the same effect, we also illustrate how the driven rings could be used as memory devices, in which the characteristic charging and discharging timescales can be tailored in order to achieve efficient writing/erasing protocols.

The paper is organized as follows. In the next Section we introduce the tight binding model describing the quantum ring connected to two metallic leads, and elucidate the numerical method employed to perform the real-time simulations. In Section III the numerical results for the non-adiabatic pumping are presented. We show two different setups to produce a pumped current by varying the external magnetic field piercing the ring, and we discuss the results in the light of a recent theory of quantum magnetic moments. An interesting application beyond pumping is proposed in Section IV, in which it is shown that the magnetically driven ring could also operate as a memory device. Finally the summary and the main conclusions are drawn in Section V.

II. MODEL AND NUMERICAL METHOD

Using the tight-binding version of the partition-free approach to quantum transport[12] we consider left (L) and right (R) leads connected to a polygonal ring with N sites in presence of an ac magnetic fied. The time-dependent model Hamiltonian is[13]:

$$H(t) = H_{\rm ring}(t) + H_L + H_R + H_T, \qquad (1)$$

where the ring Hamiltonian reads

$$H_{\rm ring}(t) = \sum_{m,n=1}^{N} h_{mn}(t) c_m^\dagger c_n \qquad (2)$$

with hopping integrals $h_{mn} = 0$ if m and n are not nearest neighbors. The time-dependent magnetic field with flux $\phi(t)$ piercing the ring is accounted for via the Peierls prescription $h_{m,n}(t) = t_h e^{\pm i \frac{2\pi\phi(t)}{N\phi_0}} \equiv e^{\pm i \frac{\alpha(t)}{N}}$ for nearest neighbors, where ϕ_0 the flux quantum and the positive sign holds if the bond $m \to n$ runs clockwise and negative if it is counterclockwise. This is one of the many ways to insert a given flux in the ring: less symmetric ways are not gauge-equivalent in the time-dependent case but the symmetric choice is the most natural. The leads are

modeled by semi-infinite tight binding chains described by the Hamiltonians $H_{L,R}$ with nearest-neighbour hopping t_h and on-site energies $\epsilon_{L,R}$. The ring is connected to the leads via a tunneling Hamiltonian H_T with hopping t_h connecting two nearest-neighbour sites of the ring denoted with A and B with the ending sites of lead L and R respectively (see e.g. Fig. 1). At equilibrium the occupation of the system is determined by the chemical potential μ, which in the rest of the work is assumed to be zero.

The numerical results below are obtained by computing the exact time evolution of the system with a finite number of sites N_{lead} in both L and R leads. We first calculate the equilibrium configuration by numerically diagonalizing the $(N_{\text{lead}} + N) \times (N_{\text{lead}} + N)$ matrix $H(0)$, and then we evolve the equal-time lesser Green's function of the system given by

$$[G^<]_{nm}(t) \equiv G^<_{nm}(t) = i\langle c^\dagger_n(t) c_m(t)\rangle. \tag{3}$$

In order to perform the time propagation we discretize the time and calculate the matrix $G^<(t)$ according to

$$G^<(t_n) \approx e^{-iH(t_n)\Delta t}\, G^<(t_{n-1})\, e^{iH(t_n)\Delta t}\,, \tag{4}$$

where $t_n = n\Delta t$, Δt is the time step, n is a positive integer and $G^<(0) = if[H(0)]$, with f the equilibrium Fermi function. The time-dependent current flowing between sites m and n connected by a bond with hopping integral h_{mn} is given by the average of the operator[14]

$$J_{mn} = -i(h_{mn} c^\dagger_m c_n - h_{nm} c^\dagger_n c_m) \tag{5}$$

and is explicitly given by

$$J_{mn}(t) = 2\text{Re}[h_{mn}\, G^<_{nm}(t)]\,. \tag{6}$$

Anagously the density at site m reads

$$n_m(t) = \text{Re}[-iG^<_{mm}(t)] \tag{7}$$

This approach allows us to reproduce the time evolution of the infinite-lead system provided N_{lead} is large enough and the the propagation times do not exceed the critical time $T_{\text{lead}} \approx N_{\text{lead}}/t_h$[15].

III. NON-ADIABATIC PUMPING

First, we consider the ring connected to identical leads and we seek a method for pumping charge form lead L to lead R (or viceversa) by employing the flux in the ring. To make a pump, we need a cycle whereby the system shifts some charge and ends with no flux in the ring, ready for starting again. If we insert some time-dependent flux $\phi(t)$ and then reduce it to zero, this is equivalent to using an electromotive force (EMF) first, e.g., clockwise and then counterclockwise. In such numerical experiments we did observe some charge shift in the wires, although the total transmitted charge at the end of the cycle is close to zero, irrespective of the detailed shape of $\phi(t)$. In Figure 1 we report the results of such a calculation for the ransmitted charge $Q_{\text{B}\to}(t) = \int_0^t J_B(\tau)d\tau$ where J_B is the current through B to the right, and similarly $Q_{\to\text{A}}(t) = \int_0^t J_A(\tau)d\tau$. We see that a positive number of electrons is going to the right and a negative one enters from the left, and finally the ring population is decreased without any significant shift of charge from left to right.

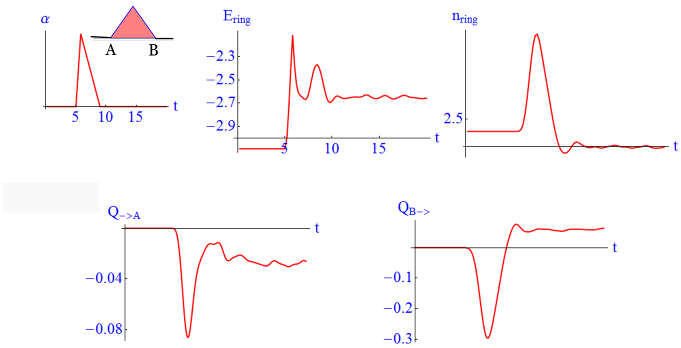

FIG. 1. Charging of a cluster with $N = 3$ sites at half filling, connected to identical wires with $\epsilon_L = \epsilon_R = 0$ by a burst of flux. Top left: time dependence of the Peierls phase $\alpha(t) = 2\pi\phi(t)/\phi_0$ inserted in the ring. The shaded area denotes where the flux is inserted. The inset shows the geometry: wires are joined to sites A and B. Top center: time dependence of the expectation value of the ring Hamiltonian, showing that the ring remains excited at the end. Top right: total electron number per spin in the ring, which remains charged at the end. Bottom left: electron number (integrated current) from left wire to A. Bottom right: electron number from B to the right wire. Charge is expelled symmetrically from the ring.

Thus in order to produce a net pumped charge, an alternative protocol is needed. One can start from zero flux and insert an integer number of flux quanta. In this way, the final Hamiltonian is equivalent to the initial one. As one can see in Figure 2 this produces the desired effect in a ring with $N = 6$, i.e. the same amount of charge is transferred at any cycle. To understand the physical origin of the charge transfer one can think of the ring as a renormalised $A - B$ bond in the circuit. By inserting the time-dependent flux in the ring, one is producing a complex effective $A - B$ bond which creates a phase difference between the L and R leads. In the time-dependent case the phase difference entails an effective level difference, equal to $\dot{\alpha}$, which is the reason why a charge transfer takes place. This effect does not decrease with the number of sides N. Actually, by repeating the calculation with the same parameters but $N = 18$ (not shown), one obtains a very similar behavior with somewhat higher steps (the staircase halts at about 3.0). However, it decreases steadily if one increases the switch-on time of the flux T_{sw}. Numerically we find that the height of the staircase in Fig.2 goes like $T_{\text{sw}}^{-0.3}$. There-

fore the charge transfer is a non-adiabatic process.

Strictly speaking, this is not yet pumping since the flux changes linearly in time and this is equivalent to a constant electromotive force inside the ring, rather than an AC one. A strict example of pumping is presented below.

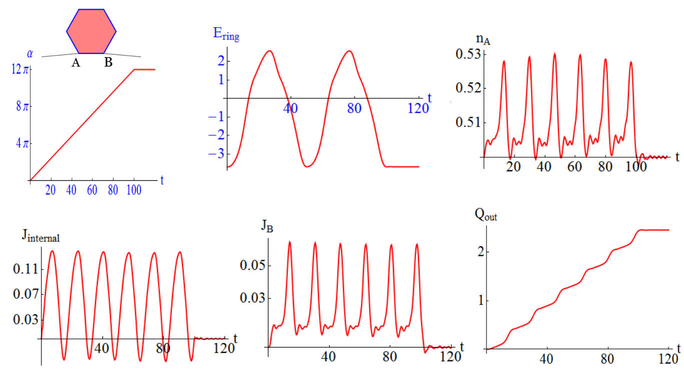

FIG. 2. (Color online) quantum pumping by a 6-atom ring at half filling, connected as shown (wires are joined to neighboring sites A and B) to identical wires and swallowing up an integer number of fluxons. The shaded area denotes where the flux is inserted. The time in abscissas is in $\frac{\hbar}{t_h}$ units. Top left: the Peierls phase $\alpha(t)$ inserted in the ring; the final value is 12π. Top center: the expectation value of the ring Hamiltonian. Top right: occupation number at site A. Bottom left: current J_{internal} flowing through all the internal bonds. Bottom center: current at A in left wire. Bottom right: *staircase* shaped increase of the electron number $Q_{\text{out}} = Q_{\rightarrow A} = -Q_{B\rightarrow}$ pumped from left to right into wires. The hight of the *staircase* decreases if the switching time increases. In other terms, the process is observed to be non-adiabatic.

We considered rings connected to wires having different equilibrium occupancy. If the atoms in the left wire have energy levels $\epsilon_L = 2$ while in the rest of the system the levels are at $\epsilon = 0$, one can expect that when the EMF tends to pump charge into the wires this will happen more effectively towards the right, because of the energy barrier $\epsilon_L - \epsilon_R$ at the junction. The phase is inserted as a series of triangular pulses, each involving much less than ϕ_0. As one can see in Figure 3, this choice of the model parameters again produces the desired effect for a ring with $N = 9$. The quasi-periodic behavior of the expectation value of the ring Hamiltonian suggests that after each cycle the ring returns to the same state. The relaxation is actually incomplete, but the oscillations of $E_{\text{ring}}(t)$ between successive flux pulses around the ground-state value are very small. However at each cycle a net charge is thrown into the external ballistic circuit. The charge injection occurs mainly during the duration of the flux pulses, and the transient effects due to incomplete relaxation do not reduce the pumping.

Symmetrically connected rings cannot have a magnetic moment, since by time reversal one changes the signs of currents and magnetic field and obtains an equivalent

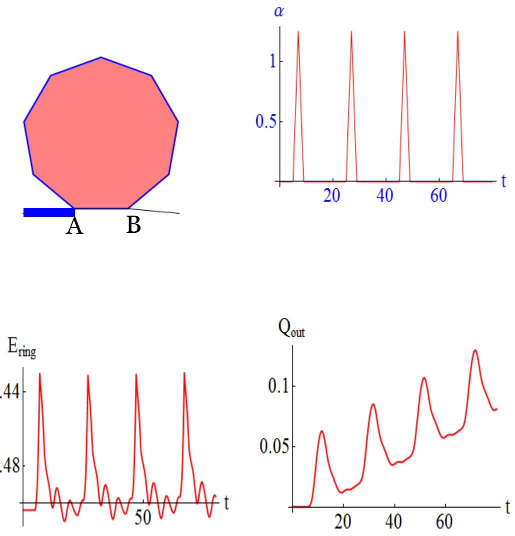

FIG. 3. (Color online) Pumping by a tight-binding ring with $N = 9$ at half filling, connected to a junction with $\epsilon_L = 2$, $\epsilon_R = 0$. Top left: sketch of the geometry (wires are joined to neighboring sites A and B and the shaded area denotes where the flux is inserted.). Top right: time dependence of the Peierls phase $\alpha(t)$. Bottom left: expectation value of the ring Hamiltonian versus time. Bottom right: increase of the electron number $Q_{\text{out}} = Q_{\rightarrow A} = -Q_{B\rightarrow}$ pumped from left to right into wires.

problem. We are considering laterally connected rings because they are maximally asymmetric. We can understand qualitatively how the device works. In the pumping process, an applied magnetic field threading the ring excites a current in the external circuit. This is the inverse of the generation of a magnetic moment by the current excited by an external bias. The two processes must be related, and this analogy led us to the present work. In a recent[11] work we showed that the magnetic moment generated by a bias-induced current is at least quadratic in the bias. In the present time-dependent problem where the roles of cause and effect are interchanged, we find that the pumped charge is quadratic in the magnetic flux. Our numerical calculations show that for any number N of sides the charge Q_{out} pumped at each cycle grows with the square of the height $\alpha_{\max}$ of the triangular pulses in Figure 3. We notice that this is typical behavior of one parameter pumping, as already found by Foa Torres[8] who considered an open dot driven by time-dependent gate voltages. If the response is linear the Brouwer theorem[5] implies adiabatic behavior and only multi-parametric pumping can be obtained[16]. The absence of a linear contribution of the ring magnetic moment in the applied bias[11] and the realization of one-parameter non-adiabatic pumping are both manifestations of the same quantum effect.

We have also studied the robustness of the above effect by varying the model parameters. As one can see in Figure 4, the pumping efficiency of the ring is optimal

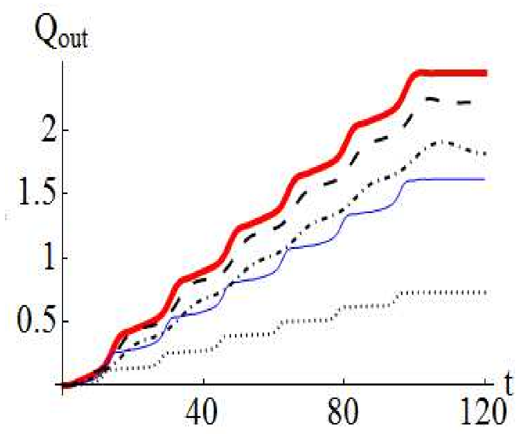

FIG. 4. (Color online) $Q_{\rm out} = Q_{\rightarrow A} = -Q_{B\rightarrow}$ versus time for the same geometry and flux as in Figure 2 but different hopping in the ring. Thick line: $t_{ring} = t_h$ as in Figure 2. Dashed line: $t_{ring} = t_h/2$. Dash-dotted line: $t_{ring} = t_h/4$. Thin line: $t_{ring} = 2t_h$. Dotted line: $t_{ring} = 4t_h$.

for $t_{\rm ring} \sim t_h$. An increase in $t_{\rm ring}$ implies that the electron prefers to move around in the ring and this causes a reduced tranferred charge. However the pumping also decreases when the hopping $t_{\rm ring}$ is reduced, because the circulating current is proportional to $t_{\rm ring}$. This is expected since at the limit $t_{\rm ring} = 0$ the circuit is interrupted, but the decrease is seen to be rather slow and for $t_{\rm ring} = t_h/4$ the pumping is still comparable to the $t_{\rm ring} = t_h$ case. Overall we may conclude that the proposed pumping is a robust effect, which survives to variations or renormalizations of the parameters such as could result from interactions in conductors which are well described by a Fermi liquid picture.

IV. MEMORY STORAGE

The possibility of pumping a desired charge form one lead to the other via a suitable protocol suggests that quantum rings can also be used as memory devices. In Figure 5 we report the outcomes of the insertion and removal of 3.5 flux quanta into a ring with $N = 17$. The on-site energies on the leads are $\epsilon_L = 2$ while $\epsilon_R = 0$. The disturbance lasts $10/t_h$ time units. After the end of the pulse the ring remains in an excited state, as one can see by the expectation value of the ring Hamiltonian, which does not return to the ground state value, thus indicating that that the ring remains charged. According to the above discussion, when the ring is connected to an asymmetric junction, a current is generated in the wires. From the bottom panels of Figure 5 one can see that the charge which is expelled to the L wire is not totally compensated by the charge coming from the R wire. If the ring eventually returns to the original charge state, it does so quite slowly compared to the time it takes to get charged. So we may say that the ring keeps memory of

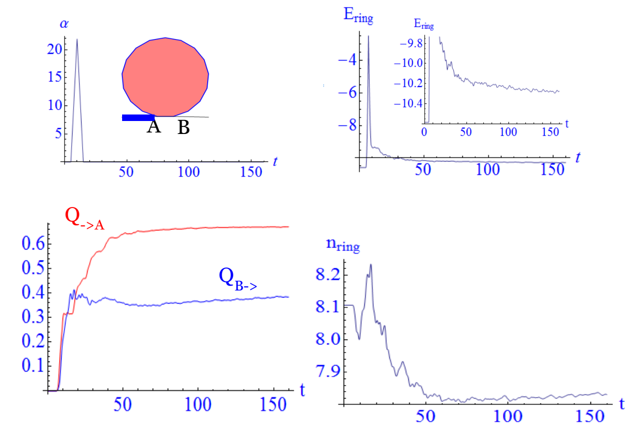

FIG. 5. (Color online) Effects of a strong magnetic pulse on a ring with $N = 17$ connected to a junction ($\epsilon_L = 2$ while $\epsilon_R = 0$). Top left: the geometry. Top center: time dependence of the phase pulse $\alpha(t)$ inserted in the shaded area. Top right: expectation value of the ring Hamiltonian. Bottom left: charge from A to the left wire. Bottom center: charge from the right wire to B. Bottom right: electron population of the ring, which remains charged, much more than in Figure 1, thus keeping a memory of the pulse.

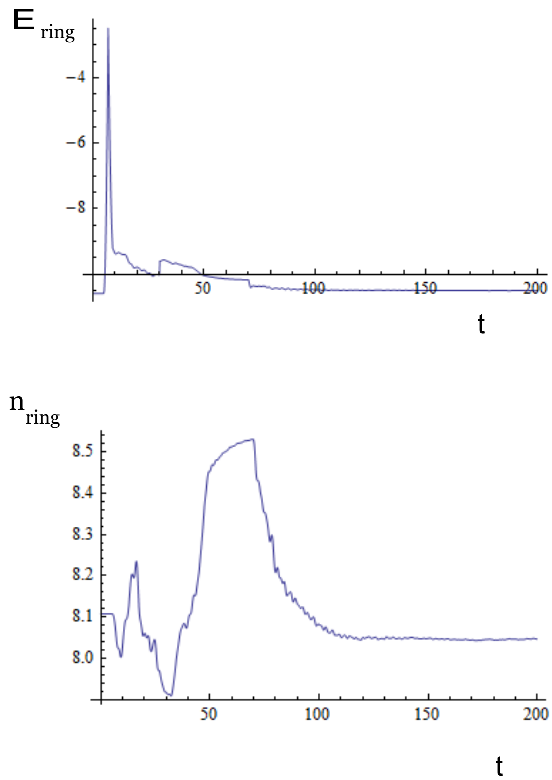

FIG. 6. (Color online) Same calculation as in Fig.5 performed in the 17-sided ring, but with the $A-B$ bond cut between times $t = 30$ and $t = 70$. The ring energy and occupation tend to return to the values they had at the beginning, and the memory of the flux is thereby erased.

the pulse. However if a memory is to be useful we must know a fast mechanism to erase it. We have observed that sending a negative pulse does not achieve that; rather, the effects tend to sum. This is not surprising, since the charge shift is a quadratic function of the flux. On the other hand we have observed flushing the ring with an external bias does not efficiently reduce the charge on it nor does it bring the ring any closer to the ground state energy. We found an efficient alternative, however, which consists in removing the $A - B$ bond for some tens of time units. The effects of doing so are shown in Figure 6: the system becomes a mere junction, so the perturbation tends to delocalize. In this way the initial condition is re-enabled, as required for a memory storage device.

V. CONCLUSIONS

In conclusion we have invstigated the possibility of single-parameter nonadiabatic pumping in a quantum circuit hosting a ballistic ring pierced by a time-varyong magnetic field. We adopted a time-dependent approach in which the electron dynamics is calculated in real time. We provide evidence that a pumped current can be generated in asymmetric geometries, as well as if integer number of flux quanta are inserted in time via a ramp protocol. Finally we have shown that driven quantum rings can also be used as memory devices with the possibility of performing basic writing/erasing operations. Our results extend those of a recently proposed quantum theory of magnetic moments in quantum rings[11], and allow us to conclude that the absence of a ring magnetic moment to first-order in the applied bias and the existence of one-parameter non-adiabatic pumping are both manifestations of the same quantum effect.

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